Astroparticle Physics, NAASPH-12

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Friedmann: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2}$

Exam, April 11, 2013.
09:00-12:00, Aletta Jacobshal01
5 problems (total of 100 points).

$a = \frac{1}{1+z}$ $\quad 1+z = \frac{1}{a}$ $\quad z = \frac{1}{a} - 1$

Write the solution of every problem on a separate piece of paper with name and student number.

Write clearly, it should be readable.

Problem 1 (25 pnts in total)
In Cosmology one distinguishes four different terms in the total energy density; namely the matter, the radiation, the vacuum, and the curvature terms.

10 pnts a. Give for each of the 4 types of energy density their scaling with red-shift z and their distinguishing features.

8 pnts b. What is the experimental evidence that a large fraction of the total energy content of the Universe has to be attributed to dark energy?

2 pnts c. How large is the fraction of dark energy?

5 pnts d. Give simple physical arguments for the fact that vacuum energy has a negative pressure.

Problem 2 (20 pnts in total)

2 pnts a. The flux of cosmic rays changes over many orders of magnitude when the energy of a cosmic ray changes. Give an approximate relation between flux and energy (or sketch the flux spectrum).

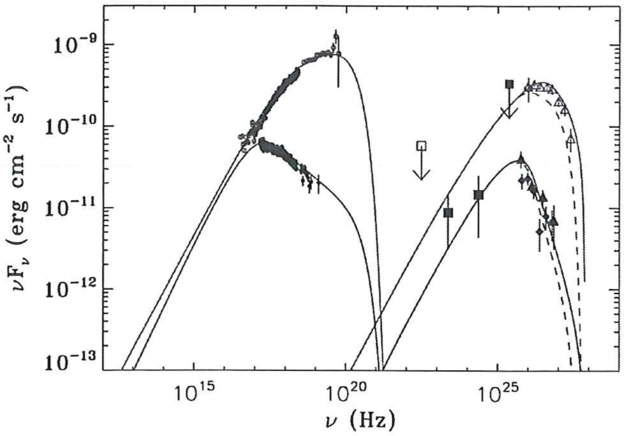
6 pnts b. Describe the generic processes which are used to model the acceleration mechanism of cosmic rays; explain their difference.

4 pnts c. Where could these processes occur in the Universe?

4 pnts d. What is the GZK process? Describe which particles interact in this process and which reaction productions can emerge from this process.

4 pnts e. What is the effect of the GZK process on the flux spectrum?

Problem 3 (20 pnts in total)
The figure displays the Spectral Energy Density (this is the product of the flux $F(\nu)$ and the frequency ν) as a function of the frequency for the extra-Galactic object Mrk501. Mrk501 has been classified as a blazar: this a very compact object (most likely a super-massive black hole, centered in an elliptical-shaped galaxy). Such objects are being regarded as possible sources of high-energy cosmic rays.



4 pnts a. The spectrum shows a bump around a frequency of 10^{19} Hz and another bump at about 10^{27} Hz. Which type of instruments could have been used for these two measurements; indicate their possible locations.

4 pnts b. Give a possible physics explanation for the bump around 10^{19} Hz and which physics parameters play an important role?

4 pnts c. Give two different physics explanations for the bump around 10^{27} Hz.

4 pnts d. Indicate for both of these two explanations the consequence for particle acceleration near this blazar.

4 pnts e. Describe a method which can be used to discriminate between these two explanations.

Problem 4 (15 pnts in total)

8 pnts a. Present the proof that the expression for the kinetic energy E_R of a nucleus of mass M_R recoiling in an elastic collision with a dark matter particle of mass M_D and incident kinetic energy E_D in terms of the angle, θ, of emission relative to the incident direction is given as:

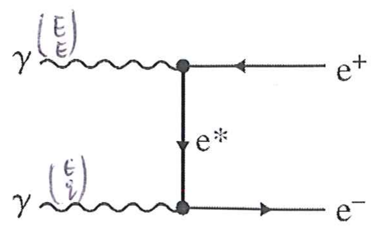
$$E_R = \frac{\mu^2 v^2}{M_R}(1 - \cos\theta)$$

where μ is the reduced mass for this collision: $\mu = M_D M_R/(M_D + M_R)$

4 pnts b. Find the limiting values of recoil energy in terms of M_D, M_R, and v.

3 pnts c. Calculate the maximum recoil energy of a nucleus of 80 proton masses, in collision with a dark matter particle of mass 1000 times the proton mass, traveling with a typical galactic velocity of 200 km s^{-1}.

Problem 5 (20 pnts in total)



The process of photo-induced lepton-pair production on a nucleus is described by the Feynman diagram to the right where we want to use it to calculate the ratio of the cross section for the production of $e^+ - e^-$ pairs versus that of $\mu^+ - \mu^-$ pairs. One of the photons is the high-energy (real) photon with energy $E_r = E$ and $\vec{q_r} = E$.

The other (virtual) photon is from the Coulomb field of the nucleus, $E_v = \epsilon$, $\vec{q_v} = q$. All momenta in this problem are taken in the $\hat{z}$-direction and we use units where $c = 1$. Since the mass of the nucleus is much larger than that of the leptons you may assume $\epsilon = 0$.

7 pnts a. Give the expression for q (in the limit that $E \gg m_\mu$) in terms of E and masses for the case of muon pair production (both muons with momentum k in the $\hat{z}$-direction). Use energy and momentum conservation in the vertices.

7 pnts b. Express the energy, E_v, and momentum, P_v of the virtual lepton in the diagram in terms of E and masses and show that $p_v^2 = E_v^2 - P_v^2$ is of the same order of magnitude as the mass of the produced leptons.

6 pnts c. Use the just obtained result to argue that muon-pair production is strongly suppressed as compared to electron-positron production.

Some numbers

Electron mass $m_e c^2 = 511\,\text{keV}$; Muon mass $m_\mu c^2 = 106\,\text{MeV}$; Pion mass $m_\pi c^2 = 140\,\text{MeV}$;
Proton mass: $m_p c^2 = 0.938\,\text{GeV}$
Conversion: $1\,\text{eV/c}^2 = 1.78 \times 10^{-36}\,\text{kg}$
Boltzmann's constant: $k = 8.62 \times 10^{-11}\,\text{MeV/K}$
Planck's constant: $h = 4.1 \times 10^{-15}\,\text{eV s}$
Avogadro's number: $N_A = 6 \times 10^{23}\,/\text{mol}$
Solar Mass: $M_\odot = 1.99 \times 10^{30}\,\text{kg}$
Parsec: $1\,\text{pc} = 3.1 \times 10^{16}\,\text{m}$
Velocity of Sun w.r.t. center Milkyway: $V_\odot = 270\,\text{km/s}$
Velocity of Earth in orbit around the Sun: $V_\oplus = 30\,\text{km/s}$
Typical galactic dark matter density: $\rho_{DM} = 9\ \text{k}\ M_\odot/\text{pc}^3$

Air-shower physics

At 10 km height the density of the atmosphere is $0.4 \times 10^{-3}\,\text{g cm}^{-3}$.
The penetration depth for pions in air is $\lambda_\pi = 120\,\text{g cm}^{-2}$, for protons $\lambda_p = 90\,\text{g cm}^{-2}$, and for iron is $\lambda_{Fe} = 5\,\text{g cm}^{-2}$.
The mean travel distance in vacuum of a pion with energy E is $d_{\pi^0} = \gamma 25 \times 10^{-9}\,\text{m}$ and $d_{\pi^\pm} = \gamma 7.8\,\text{m}$ where the relativistic γ-factor is given by $\gamma = E/mc^2$ and $m_\pi c^2 = 140\,\text{MeV}$.

Integrals

For $c > 0$ we have:

$$\int_0^\infty c\,e^{-cx}dx = 1\,; \quad \int_0^\infty c\,x\,e^{-cx}dx = 1/c\,; \quad \int_0^\infty c\,x^2 e^{-cx}dx = 2/c^2$$

$(1+x)^\alpha \approx 1+\alpha x$